

Exact Solution to the “Auxiliary Extra Dimension” Model of Massive Gravity

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Abstract

The “Auxiliary Extra Dimension” model was proposed in order to provide a geometrical interpretation to modifications of general relativity, in particular to non-linear massive gravity. In this context, the theory was shown to be ghost free to third order in perturbations, in the decoupling limit. In this work, we exactly solve the equation of motion in the extra dimension, to obtain a purely 4-dimensional theory. Using this solution, it is shown that the ghost appears at the fourth order and beyond. We explore potential modifications to address the ghost issue and find that their consistent implementation requires going beyond the present framework.

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1 Introduction and Summary

While general relativity is a geometric theory, the same cannot be said of models of massive gravity. Moreover, it has been difficult to construct such massive models that are free of ghost instabilities [1, 2, 3]. The “auxiliary extra dimension” (AuXD) model was proposed in an attempt to address both these issues simultaneously [4, 5]. In this model the mass term arises from the extrinsic curvature of the 4-dimensional spacetime in 5 dimensions, the 5th dimension being non-dynamical. Thus in this model massive gravity acquires a geometric interpretation. Subsequently, it was verified in [6] that this theory was ghost free to 3rd order in the “decoupling limit”¹.

Since then, 4-dimensional theories of massive gravity that are potentially ghost free to all orders have been constructed [7, 8] and their systematics explored [10]. In fact, the AuXD model seems to have been a motivation for revisiting the earlier works [11, 12] which led to the new developments. It is therefore appropriate to determine where this model belongs in the new scheme of things.

The AuXD model leads to two coupled equations: (i) a purely 4 dimensional Einstein-Hilbert equation with extrinsic curvature contributions, and (ii) an equation for the extra dimension u . The u -equation converts the extrinsic curvature contributions to mass terms. So far, this equation has been solved perturbatively to third order. It leads to a 4 dimensional massive gravity that is ghost free to this order in the decoupling limit [6]. In the present paper, we solve the u -equation *exactly* to obtain a purely 4-dimensional, closed-form expression for the non-linear mass term. This allows us to compare the AuXD model to the recently constructed theories of massive gravity [7, 8, 10] and examine its stability to any order. We find that in the standard interpretation of the model, the ghost re-enters at the 4th order, and hence the theory is not consistent. There exist non-standard modifications that can potentially alleviate the ghost problem to any given order in the decoupling limit. However, we show that such modifications cannot be consistently implemented within the present setup. (However, see footnote 6).

The paper is organized as follows: The AuXD model is introduced in section 2. In section 3 we solve the u -equation in terms of integration constants. These are determined in section 4 in terms of boundary conditions. There we obtain the 4-dimensional massive gravity action and its equations of motion. In section 5 this model is compared to the potentially ghost free massive actions constructed recently and a ghost is shown to appear at the fourth order and beyond. We also explore modifications of the boundary condition in an attempt to resolve the ghost issue.

2 The Auxiliary Extra-dimension Model

The starting point is the 4-dimensional Einstein-Hilbert action with an extrinsic curvature term involving the “auxiliary extra-dimension” u and $\tilde{g}_{\mu\nu}(x, u)$ with $\mu, \nu = 0, 1, 2, 3$ [4, 5],

$$S = -M_p^2 \int d^4x \left[\sqrt{g} R + \frac{m^2}{2} \int_{-1}^{+1} du \sqrt{\tilde{g}} (k_{\mu\nu} k^{\mu\nu} - k^2) \right] + S_{\text{matter}}. \quad (1)$$

¹The decoupling limit corresponds to taking the graviton mass $m \rightarrow 0$ and $M_p \rightarrow \infty$ while keeping $m^2 M_p$ fixed. In practice, this means retaining terms to first order in the metric perturbation $h = g - \eta$, but to all orders in the non-linear Stückelberg fields π . Thus this limit allows one to consider the non-linearities that are most relevant to the ghost problem. It is mostly in this limit that the newly constructed theories of massive gravity are shown to be ghost free [7, 8]. In these models, the absence of the ghost to all orders away from the decoupling limit is still an open question (see, for example, [9]). The ghost analysis of the AuXD model in the present paper is always done in the decoupling limit.

Here, $k_{\mu\nu} = \frac{1}{2}\partial_u \tilde{g}_{\mu\nu}$, $k = \tilde{g}^{\mu\nu} k_{\mu\nu}$, and the 4-dimensional metric is $g_{\mu\nu}(x) = \tilde{g}_{\mu\nu}(x, 0)$. The matter action S_{matter} is also localized at $u = 0$.

To obtain the equations of motion, the action is varied with respect to $\tilde{g}^{\mu\nu}(x, u)$. Then for $u \neq 0$ one obtains the “ u -equation”,

$$\frac{1}{\sqrt{\tilde{g}}}\partial_u \left(\sqrt{\tilde{g}}[k^{\mu\nu} - k\tilde{g}^{\mu\nu}] \right) - \frac{1}{2}\tilde{g}^{\mu\nu}(k_{\rho\sigma}k^{\rho\sigma} - k^2) + 2(k^\mu{}_\lambda k^{\lambda\nu} - k k^{\mu\nu}) = 0. \quad (2)$$

Solving this requires specifying boundary conditions $\tilde{g}_{\mu\nu}(x, 0) = g_{\mu\nu}(x)$ and $\tilde{g}_{\mu\nu}(x, 1) = f_{\mu\nu}(x)$, for some $f_{\mu\nu}$. Furthermore, integrating the variation over $u \in \{-\epsilon, \epsilon\}$ and assuming reflection symmetry about $u = 0$, one obtains the 4-dimensional equation of motion,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + m^2(k_{\mu\nu} - g_{\mu\nu}k) \Big|_{u=0^+} = G_N T_{\mu\nu}. \quad (3)$$

For $\tilde{g}_{\mu\nu}(x, u) = \eta_{\mu\nu} + (1 - u)h_{\mu\nu}(x)$, at the linearized level, one recovers the Fierz-Pauli massive gravity [4, 5]. In [6] the model was analyzed to cubic order and was shown to be free of ghosts to that order. This supported the proposal that the action (1) provided a consistent non-linear generalization of the Fierz-Pauli mass. The use of the extra-dimension u can then be understood as a way of packaging the non-linearities. The flip side is that the non-linear structure of the mass term is not explicit in the model. Below, we solve for the u -dependence to obtain the purely 4-dimensional form of the mass term. Then, the massive theory can be written entirely in 4-dimensions with no reference to the extra-dimension.

Before proceeding let us point out that the action (1) is invariant only under 4 dimensional general coordinate transformations that do not involve u . Being u -independent, these transform $\tilde{g}_{\mu\nu}(u = 0) = g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}(u = 1) = f_{\mu\nu}$ in the same way. Hence $f_{\mu\nu}$ transforms as a rank 2 tensor and can be fixed to a specific form only in a given gauge.

3 Solution of the u -Equation

Using $\partial_u \sqrt{\tilde{g}} = \sqrt{\tilde{g}} k$ and $k^\mu{}_\nu = \frac{1}{2}\tilde{g}^{\mu\sigma}\partial_u \tilde{g}_{\sigma\nu}$, the u -equation (2) becomes

$$\partial_u(k^\mu{}_\nu - k\delta^\mu_\nu) + k(k^\mu{}_\nu - k\delta^\mu_\nu) - \frac{1}{2}\delta^\mu_\nu(k^\rho{}_\sigma k^\sigma{}_\rho - k^2) = 0. \quad (4)$$

We introduce the notation \mathbb{k} for a matrix with elements $k^\mu{}_\nu$ and denote its traceless part by \mathbb{k}_t and its trace by k . In terms of these the u -equation splits into,

$$\partial_u \mathbb{k}_t = -k \mathbb{k}_t, \quad (5)$$

$$\partial_u k + \frac{1}{2}k^2 + \frac{2}{3}\text{Tr}(\mathbb{k}_t^2) = 0. \quad (6)$$

Eliminating $\text{Tr}(\mathbb{k}_t^2)$ from the above leads to a second order equation $\partial_u^2 k + 3k\partial_u k + k^3 = 0$, with the solution,

$$k(u) = \frac{2(u + c)}{(u + c)^2 - d^2} = \partial_u \ln[(u + c)^2 - d^2]. \quad (7)$$

The equation for \mathbb{k}_t can now be integrated to,

$$\mathbb{k}_t = \mathbb{C}_t [(u + c)^2 - d^2]^{-1}. \quad (8)$$

Here, c , d and the traceless matrix \mathbb{C}_t are integration constants to be determined in terms of the boundary data $g_{\mu\nu}(x)$ and $f_{\mu\nu}(x)$. Demanding that these also solve the first order equation (6), determines d as,

$$d^2 = \frac{1}{3} \text{Tr}(\mathbb{C}_t^2). \quad (9)$$

The extrinsic curvature k^μ_ν can now be reconstructed as (in matrix notation),

$$\mathbb{k} = \mathbb{k}_t + \frac{1}{4} \mathbb{1} k = \frac{\mathbb{C}_t + \frac{1}{2}(u+c)\mathbb{1}}{(u+c)^2 - d^2}. \quad (10)$$

Now, the relation $k = \partial_u(\ln \sqrt{\tilde{g}})$ that follows from the definition of $k_{\mu\nu}$ can be integrated, with the boundary condition $\det \tilde{g}(u=0) = \det g$, to give

$$\sqrt{\tilde{g}(u)} = \frac{(u+c)^2 - d^2}{c^2 - d^2} \sqrt{g}. \quad (11)$$

These solutions can be used to perform the u -integral in (1) and obtain a purely 4-dimensional action in terms of the integration constants c and d . Using the imposed Z_2 symmetry about $u=0$, and solutions (7), (10) and (11), the u -integral in (1) becomes,

$$I \equiv 2 \int_0^{+1} du \sqrt{\tilde{g}} (k_{\mu\nu} k^{\mu\nu} - k^2) = -\sqrt{g} \frac{6}{c^2 - d^2} \int_0^1 du. \quad (12)$$

Note that for the solution of the u -equation the integrand has become completely u independent! The metric equation of motion (3) in terms of integration constants, is,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + m^2 \frac{1}{c^2 - d^2} \left(g_{(\mu\lambda} \mathbb{C}_{t\nu)}^\lambda - \frac{3}{2} c g_{\mu\nu} \right) = G_N T_{\mu\nu}. \quad (13)$$

4 The 4-Dimensional Action

We first determine the integration constants c , d , and \mathbb{C}_t in terms of the boundary data $g_{\mu\nu}$ and $f_{\mu\nu}$ at $u=0$ and $u=1$. Using the u -independent invertible matrix $f_{\mu\nu}$ let us define,

$$E^\mu_\nu(u) = \tilde{g}^{\mu\lambda}(u) f_{\lambda\nu}, \quad (14)$$

or in matrix notation, $\mathbb{E} = \tilde{\mathbb{g}}^{-1} \mathbb{f}$. In terms of this variable we have,

$$\mathbb{k}(u) = -\frac{1}{2} \mathbb{E}^{-1} \partial_u \mathbb{E} = -\frac{1}{2} \partial_u \ln \mathbb{E}(u). \quad (15)$$

Of course, the last equality does not hold for a generic matrix $\mathbb{E}(u)$. In the present case this is valid only because the u -dependence of \mathbb{k} in (10) is not contained in its matrix structure, but rather in the scalar coefficients of the commuting matrices \mathbb{C}_t and $\mathbb{1}$. These will also determine the matrix structure of \mathbb{E} ensuring that $[\mathbb{E}, \partial_u \mathbb{E}] = 0$. \mathbb{E} (with an upper and a lower index) is introduced so that power series expansions are defined unambiguously. Also, in the above construction, interchanging $f_{\mu\nu}$ and $g_{\mu\nu}$ will lead to the same eventual outcome. Integrating this, using the solution for \mathbb{k} (10) gives,

$$\ln \mathbb{E}(u) - \ln \mathbb{E}(0) = -2 \int_0^u \mathbb{k} du = \mathbb{D}(u) - \mathbb{D}(0), \quad (16)$$

where ²,

$$\mathbb{D}(u) = \frac{\mathbb{C}_t}{d} \ln \left[\frac{d+c+u}{d-c-u} \right] - \frac{1}{2} \ln [(u+c)^2 - d^2] . \quad (17)$$

Specifically, for $u = 1$, where, $\tilde{\mathfrak{g}}(1) = \mathfrak{f}$, one has $\ln \mathbb{E}(1) = 0$, leading to,

$$\ln(\mathfrak{g}^{-1}\mathfrak{f}) = \frac{\mathbb{C}_t}{d} \ln \left[\frac{d-(c+1)}{d+c+1} \frac{d+c}{d-c} \right] - \frac{1}{2} \ln \left[\frac{c^2 - d^2}{(c+1)^2 - d^2} \right] . \quad (18)$$

This determines the integration constants. The trace part gives (using $\text{Tr} \ln \mathbb{E} = \ln \det \mathbb{E}$),

$$\sqrt{\det(\mathfrak{g}^{-1}\mathfrak{f})} = \frac{(c+1)^2 - d^2}{c^2 - d^2} , \quad (19)$$

which is equation (11) for $u = 1$. To solve the traceless equation, introduce $\mathbb{L} = \ln(\mathfrak{g}^{-1}\mathfrak{f})$ and its traceless part \mathbb{L}_t ,

$$\mathbb{L}_t = \ln(\mathfrak{g}^{-1}\mathfrak{f}) - \frac{1}{4} \text{Tr} \ln(\mathfrak{g}^{-1}\mathfrak{f}) . \quad (20)$$

Then,

$$\mathbb{L}_t = \frac{\mathbb{C}_t}{d} \ln \left[\frac{d-(c+1)}{d+c+1} \frac{d+c}{d-c} \right] . \quad (21)$$

On squaring, tracing and using (9) one gets,

$$e^{\sqrt{\frac{1}{3} \text{Tr}(\mathbb{L}_t^2)}} = \frac{d-(c+1)}{d+c+1} \frac{d+c}{d-c} , \quad (22)$$

where, for later reference,

$$\text{Tr}(\mathbb{L}_t^2) = \text{Tr} [\ln(\mathfrak{g}^{-1}\mathfrak{f})]^2 - \frac{1}{4} [\text{Tr} \ln(\mathfrak{g}^{-1}\mathfrak{f})]^2 . \quad (23)$$

From these one finds that,

$$\mathbb{C}_t = \frac{d \mathbb{L}_t}{\sqrt{\frac{1}{3} \text{Tr}(\mathbb{L}_t^2)}} . \quad (24)$$

Multiplying and dividing (19) and (22) leads to,

$$\frac{1}{c \pm d} = [\det(\mathfrak{g}^{-1}\mathfrak{f})]^{1/4} e^{\mp \frac{1}{2} \sqrt{\frac{1}{3} \text{Tr}(\mathbb{L}_t^2)}} - 1 . \quad (25)$$

We can now use these expressions to write a 4-dimensional action and equation of motion entirely in terms of g and f . For the action, the relevant quantity is,

$$\frac{1}{3} F(\mathfrak{g}^{-1}\mathfrak{f}) \equiv \frac{1}{c^2 - d^2} = [\det(\mathfrak{g}^{-1}\mathfrak{f})]^{\frac{1}{2}} - 2 [\det(\mathfrak{g}^{-1}\mathfrak{f})]^{\frac{1}{4}} \cosh \left(\frac{1}{2\sqrt{3}} \sqrt{\text{Tr}(\mathbb{L}_t^2)} \right) + 1 . \quad (26)$$

Then the non-linear action with the mass term (12) becomes,

$$S = -M_p^2 \int d^4x \sqrt{-g} [R(g) - m^2 F(\mathfrak{g}^{-1}\mathfrak{f})] + S_{\text{matter}} . \quad (27)$$

This has the generic structure of a non-linear massive gravity action. Unsurprisingly, the boundary metric $f_{\mu\nu}$ has become the auxiliary metric needed to formulate massive gravity (see, for example,

²Using $\int [(u+c)^2 - d^2]^{-1} du = -\tanh^{-1}(\frac{c+u}{d})/d + \text{const.}$ and $\tanh^{-1}(x) = \frac{1}{2} \ln[(1+x)/(1-x)]$, which is valid for $|x| \leq 1$.

[10]). Note that this mass term can be written equivalently in terms of $\mathbb{f}^{-1}\mathbb{g}$ with an appropriate sign flip in the exponent of the determinants. The term $\text{Tr}(\mathbb{L}_t^2)$ (23) will look the same either way. The metric equation of motion (13) can be fully expressed in terms of $\mathbb{g}^{-1}\mathbb{f}$ using,

$$\frac{1}{c-d} + \frac{1}{c+d} = \frac{2c}{c^2-d^2}, \quad \frac{1}{c-d} - \frac{1}{c+d} = \frac{2d}{c^2-d^2}. \quad (28)$$

One then obtains,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + m^2 \left[[\det(\mathbb{g}^{-1}\mathbb{f})]^{\frac{1}{4}} \sinh \left(\frac{1}{2\sqrt{3}} \sqrt{\text{Tr}(\mathbb{L}_t^2)} \right) \left[\frac{1}{3} \text{Tr}(\mathbb{L}_t^2) \right]^{-\frac{1}{2}} \right] g_{(\mu\lambda} \mathbb{L}_{t\nu)}^{\lambda} \\ - \frac{3}{2}m^2 \left[[\det(\mathbb{g}^{-1}\mathbb{f})]^{\frac{1}{4}} \cosh \left(\frac{1}{2\sqrt{3}} \sqrt{\text{Tr}(\mathbb{L}_t^2)} \right) - 1 \right] g_{\mu\nu} = G_N T_{\mu\nu}. \quad (29)$$

This equation can also be obtained directly by varying the 4-dimensional action (27) with respect to $g_{\mu\nu}$, for fixed $f_{\mu\nu}$.

5 The Status of the Ghost Problem

The above solution is valid for any fixed $f_{\mu\nu}$. In order to obtain the Fierz-Pauli Lagrangian for massive gravity at lowest order in the fields, the standard approach is to take $f_{\mu\nu}$ to be flat [4, 5, 6]. With this premise, one can now verify that the action (27) contains a Fierz-Pauli mass and check if it can avoid the ghost instability. We show that for the standard interpretation of $f_{\mu\nu}$ the theory is not ghost free. A non-standard interpretation is also discussed below.

In the standard massive gravity context, $f_{\mu\nu}$ is the coordinate transform of the flat metric,

$$f_{\mu\nu} = \eta_{ab} \partial_\mu \phi^a \partial_\nu \phi^b. \quad (30)$$

Let us introduce the (1,1) tensor H^μ_ν so that,

$$\mathbb{g}^{-1}\mathbb{f} = \mathbb{1} - \mathbb{H}, \quad (31)$$

where as usual \mathbb{H} denotes the matrix with elements H^μ_ν . In [6], the most general, potentially ghost free, massive action was written to quintic order as a polynomial in \mathbb{H} , and with free parameters c_3, d_5 – the expression that multiplies f_7 vanishes in 4 dimensions [10]. Now, expanding the mass term in (27) to 5th order in \mathbb{H} one obtains,

$$F(g^{-1}f) = -\frac{1}{4} \left\{ \left[-(\text{Tr } \mathbb{H})^2 + \text{Tr } \mathbb{H}^2 \right] + \frac{1}{4} \left[(\text{Tr } \mathbb{H})^3 - 5 \text{Tr } \mathbb{H} \text{Tr } \mathbb{H}^2 + 4 \text{Tr } \mathbb{H}^3 \right] \right. \\ + \frac{1}{12^2} \left[-5(\text{Tr } \mathbb{H})^4 + 58(\text{Tr } \mathbb{H})^2 \text{Tr } \mathbb{H}^2 - 53(\text{Tr } \mathbb{H}^2)^2 - 132 \text{Tr } \mathbb{H} \text{Tr } \mathbb{H}^3 + 132 \text{Tr } \mathbb{H}^4 \right] \\ + \frac{1}{24^2} \left[2(\text{Tr } \mathbb{H})^5 - 41(\text{Tr } \mathbb{H})^3 \text{Tr } \mathbb{H}^2 + 123 \text{Tr } \mathbb{H} (\text{Tr } \mathbb{H}^2)^2 + 160(\text{Tr } \mathbb{H})^2 \text{Tr } \mathbb{H}^3 \right. \\ \left. \left. - 304 \text{Tr } \mathbb{H}^2 \text{Tr } \mathbb{H}^3 - 420 \text{Tr } \mathbb{H} \text{Tr } \mathbb{H}^4 + 480 \text{Tr } \mathbb{H}^5 \right] + O(\mathbb{H}^6) \right\}. \quad (32)$$

The quadratic expression is the Fierz-Pauli mass. For the choice $c_3 = \frac{1}{4}$, the cubic terms match with the corresponding terms in [7] that are ghost free in the decoupling limit, as was first shown in [6]. However, at the quartic order and beyond, no value of d_5 in [7] can reproduce the corresponding

terms here. This implies that the AuXD model is not ghost free beyond the cubic order^{3,4}. In fact, the closest ghost free expression corresponds to $d_5 = -5/12^2$ which gives the quartic term coefficients $\{-5, 57, -51, -130, 129\}$ rather than the $\{-5, 58, -53, -132, 132\}$ found above.

Can the ghost problem be cured? To identify the AuXD model with massive gravity we required $f_{\mu\nu}$ to be flat. This was consistent with the potentially ghost-free theory to cubic order. One may consider more general $f_{\mu\nu}$ to attempt to resolve the ghost issue.

In particular, one may regard $f_{\mu\nu}$ as an arbitrary function of both $g_{\mu\nu}$ and the matrix given by the right hand side of (30)⁵. With this new interpretation of $f_{\mu\nu}$ we can write,

$$\mathfrak{g}^{-1}\mathbb{f} = \mathbb{1} - M(\mathbb{H}), \quad (33)$$

where \mathbb{H} is defined such that $H_{\mu\nu} = g_{\mu\nu} - \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$. Now, one can take the equations of motion (29) for such a choice of $f_{\mu\nu}$ and identify them with the equations of motion for the potentially ghost-free actions, for example equation (4.20) in ref. [10] where the auxiliary metric there is taken to be flat. We can use this as a definition of \mathbb{f} , or equivalently of M , in (33). However, while such a procedure guarantees that the equation of motion has the correct ghost-free structure (at least in the sense of [11, 12, 7]), the resulting equation can no longer be regarded as the equation of motion for the AuXD model. This is because the boundary metric \mathbb{f} is now a function of \mathfrak{g} while the equations of motion were derived assuming that \mathfrak{g} did not vary on the boundary.

In other words, deriving the u -equation (2) from the the AuXD action (1) required setting to zero a boundary term,

$$\sqrt{\tilde{g}} (k_\nu^\mu - k\delta_\nu^\mu) \tilde{g}^{\nu\lambda} \delta\tilde{g}_{\lambda\mu}|_{u_0}. \quad (34)$$

In the standard interpretation, the boundary is at $u_0 = \pm 1$, and there, $\tilde{g}_{\mu\nu}(u = \pm 1) = f_{\mu\nu}$. For an $f_{\mu\nu}$ independent of $g_{\mu\nu}$, this is consistent with $\delta\tilde{g} = 0$. However, if $f_{\mu\nu}$ depends on $g_{\mu\nu}$, then this variation does not vanish. From our solution it is also clear that the coefficient of $\delta\tilde{g}$ does not vanish at $u = 1$. Thus the boundary term (34) is not zero.

Another option is to derive the u -equation for $u_0 = \pm\infty$, assuming that the boundary term vanishes there, but find a solution subject to the boundary condition at $u = 1$. However, even in this case, our solutions determine the behavior of \tilde{g} beyond $u = 1$. In fact, from (15), $\ln \mathbb{E} = \mathbb{D}(u) - \mathbb{D}(1)$ and then it is not difficult to see that,

$$\lim_{u \rightarrow \infty} \tilde{g}_{\mu\nu}(u) = u A_{\mu\nu}(x), \quad (35)$$

Here $A_{\mu\nu}$ is a u -independent matrix that depends on $g_{\mu\nu}(x)$. Thus, in this limit, a variation δg at $u = 0$ induces a variation $\delta\tilde{g}$ at $u = \infty$. Moreover, in this limit, $k_\nu^\mu = u^{-1}\delta_\nu^\mu$ and therefore, the coefficient of $\delta\tilde{g}$ in the boundary term becomes a u -independent non-zero factor. Hence we see that this non-standard interpretation of $f_{\mu\nu}$ is not consistent with the variation principle that gives the u -equation. To implement such a non-standard interpretation of f in AuXD model, one will have to modify the action to make it consistent with the variation principle⁶.

³The presence of the ghost can also be expected on general grounds. As stressed in [10], the potentially ghost free models of [7] have a more natural expression in terms of $\mathfrak{g}^{-1}\mathbb{f}$, rather than $\mathbb{f}^{-1}\mathfrak{g}$. This is not true of (27) which treats $g_{\mu\nu}$ and $g^{\mu\nu}$ more symmetrically, a property traced back to the structure of k_ν^μ .

⁴After the completion of this work, we were informed by C. de Rham and G. Gabadadze that the extension of their third order calculation [6] to fourth order leads to the same conclusion.

⁵We would like to thank G. Gabadadze for suggesting this possibility.

⁶However, it was pointed out more recently in [13] that the boundary term (34) modifies only eqn. (3) and not the u -equation (2). This keeps our solution (27) unchanged. In this way, [13] was able to tune the mass term order by order.

It is also possible that adding specific higher order curvature terms, like $K_{\mu\nu}^3$, to the action can address the ghost issue. However, all these approaches to the problem require foreknowledge of the ghost-free massive gravity action, contrary to the initial approach of [4, 5] in which the ghost-free structure emerged naturally to cubic order. It is not obvious that such modifications, even if implemented consistently, would still preserve the geometric interpretation of the mass term which was a virtue of the original AuXD model of [4, 5].

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